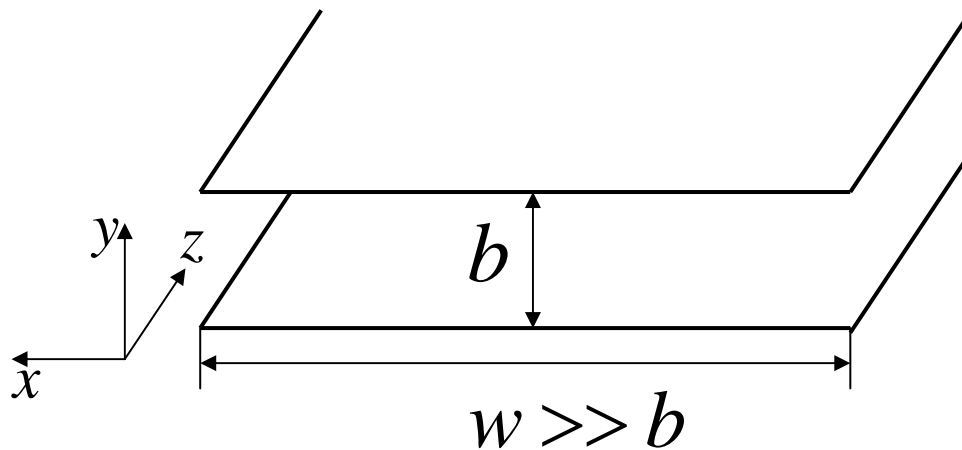


# Lect. 10: Metallic Waveguide

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Can we send EM waves without worrying about diffraction?

Consider Metallic Waveguide



TEM Solutions

$$\bar{E} = \bar{y}E_0e^{-j\beta z}$$

$$\bar{H} = -\bar{x}H_0e^{-j\beta z}$$

# Lect. 10: Metallic Waveguide

## TM<sub>m</sub> Solutions

$$E_x(y, z) = 0$$

$$E_y(y, z) = -\frac{j\beta}{n\pi/b} A_m \cos\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$E_z(y, z) = A_m \sin\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$H_x(y, z) = A_m \frac{j\omega\epsilon}{n\pi/b} \cos\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$H_y(y, z) = 0$$

$$H_z(y, z) = 0$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{b}\right)^2}$$

Quantization of  $\beta$ .  
Each  $m$  corresponds  
to one mode.

## TE<sub>m</sub> solutions

$$E_x(y, z) = \frac{j\omega\mu}{m\pi/b} B_m \sin\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$E_y(y, z) = 0$$

$$E_z(y, z) = 0$$

$$H_x(y, z) = 0$$

$$H_y(y, z) = \frac{j\beta}{m\pi/b} B_m \sin\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$H_z(y, z) = B_m \cos\left(\frac{m\pi}{b} y\right) e^{-j\beta z}$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{b}\right)^2}$$

# Lect. 10:

TM<sub>1</sub>

$$E_z(y, z, t) = A_1 \sin\left(\frac{\pi}{b} y\right) \cos(\omega t - \beta z)$$

$$E_x(y, z, t) = 0$$

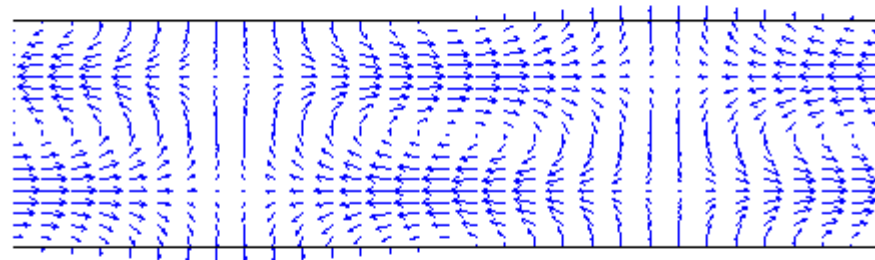
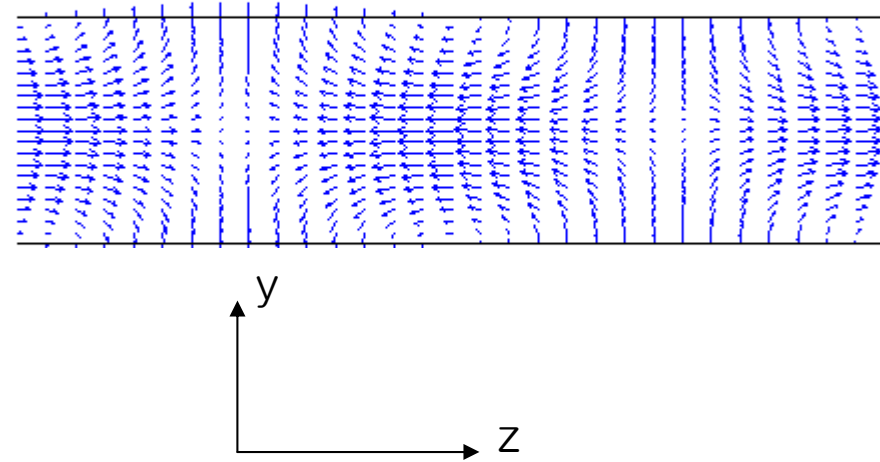
$$E_y(y, z, t) = \frac{\beta b}{\pi} A_1 \cos\left(\frac{\pi}{b} y\right) \sin(\omega t - \beta z)$$

TM<sub>2</sub>

$$E_z(y, z, t) = A_2 \sin\left(\frac{2\pi}{b} y\right) \cos(\omega t - \beta z)$$

$$E_x(y, z, t) = 0$$

$$E_y(y, z, t) = \frac{\beta b}{2\pi} A_2 \cos\left(\frac{2\pi}{b} y\right) \sin(\omega t - \beta z)$$



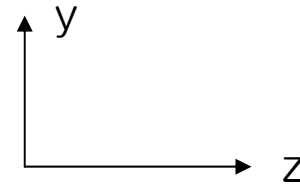
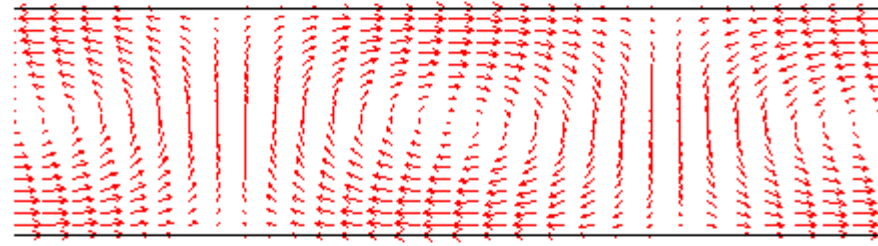
# Le

TE<sub>1</sub>

$$H_x(y, z, t) = 0$$

$$H_y(y, z, t) = -\frac{\beta}{\pi/b} B_1 \sin\left(\frac{\pi}{b} y\right) \sin(\omega t - \beta z)$$

$$H_z(y, z, t) = B_1 \cos\left(\frac{\pi}{b} y\right) \cos(\omega t - \beta z)$$

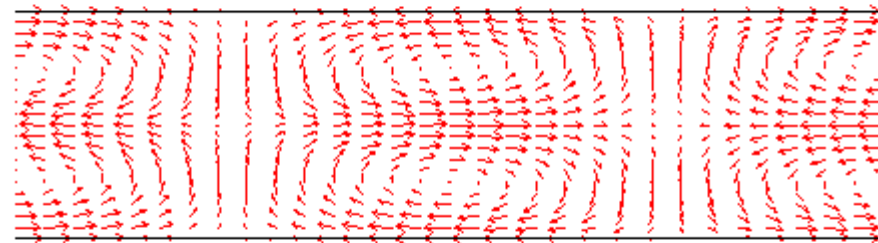


TE<sub>2</sub>

$$H_x(y) = 0$$

$$H_y(y) = -\frac{\beta}{2\pi/b} B_2 \sin\left(\frac{2\pi}{b} y\right) \sin(\omega t - \beta z)$$

$$H_z(y) = B_2 \cos\left(\frac{2\pi}{b} y\right) \cos(\omega t - \beta z)$$



# Lect. 10: Metallic Waveguide

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TM and TE difference?

TM: Transverse Magnetic => Parallel Polarization

TE: Transverse Electric => Perpendicular Polarization

# Lect. 10: Metallic Waveguide

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– Why sin, cos dependence on  $y$ ?

$$e^{jk_y y} \pm e^{-jk_y y} \sim \cos(k_y y), \sin(k_y y)$$

$$\implies k_y = k \cos \theta = \frac{m\pi}{b}$$

– Why quantization in  $k_y$  or  $\theta$ ? ( $\rightarrow$  mode)

In  $y$ -direction, wave goes through the periodic movement.

$$e^{-jk_y b} (-1) e^{-jk_y b} (-1) = e^{-j2k_y b} = 1$$

$$\therefore 2k_y b = 2m\pi \text{ and } k_y = \frac{m\pi}{b}, \beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{b}\right)^2}$$

# Lect. 10: Metallic Waveguide

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- How many modes can a given waveguide support?

$$\text{From } k_y = \frac{m\pi}{b}, nk_0 \cos \theta = \frac{m\pi}{b} \text{ and } \beta = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{b}\right)^2}$$

As  $m$  increases,

$\theta$  decreases until it becomes zero, and  $\beta$  goes down until it become zero.

$$\Rightarrow nk_0 > \frac{m\pi}{b} \quad \therefore m \text{ (integer)} < \frac{2nb}{\lambda}$$

# Lect. 10: Metallic Waveguide

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- For a given waveguide mode, what is the min. frequency that can propagate?

$$\text{From } \beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{b}\right)^2}$$

as  $\omega$  decreases,  $\beta$  decreases until it becomes zero.

$$\Rightarrow \omega^2 \mu \epsilon = \left(\frac{m\pi}{b}\right)^2$$

$$\therefore \omega_{\min} = \frac{m\pi}{b\sqrt{\mu\epsilon}} \text{ or } f_{\min} = \frac{m}{2b\sqrt{\mu\epsilon}}; \text{ Cut-off Frequency}$$